Qualifying Exam   Complex Analysis   May, 2022

1. Prove that for any $z_1, z_2 \in \{z \in \mathbb{C} : \operatorname{Re} z < 0\}$, $|e^{z_1} - e^{z_2}| \leq |z_1 - z_2|$.

2. Let f be a non-constant meromorphic function on $\mathbb{C}$ with P being the set of its poles. Prove that $f(\mathbb{C} \setminus P)$ is dense in $\mathbb{C}$.

3. Let (f_n) be a sequence of conformal maps defined on a complex domain D. Suppose $f_n \to f$ uniformly on each compact subset of D. Prove that f is either constant or a conformal map.
   Remark. A conformal map is a holomorphic and injective map. A complex domain is a nonempty connected open subset of $\mathbb{C}$.

4. Let z_0 be a pole of f. Prove that z_0 is an essential singularity of e^f.

5. Compute
$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)(x^2+4)}\, dx.$$